Il Tolomeo nelle Americhe

Lucio Russo è l'autore di un fortunato libro intitolato *L'America dimenticata. I rapporti tra le civiltà e un errore di Tolomeo* (Milano, 2013) con il quale ha inteso dimostrare che lo scienziato alessandrino si era sbagliato nel comprimere la circonferenza della Terra calcolata da Eratostene in 252.000 stadi, pari a 700 per grado. "Tolomeo non attribuisce alla circonferenza terrestre i 252.000 stadi di lunghezza che Eratostene aveva misurato e Ipparco accettato, ma solo 180.000. La lunghezza di ogni grado di meridiano passa così da 700 a 500 stadi" (*L'America dimenticata cit*., p. 151). L'inconsistenza dell'assunto di Russo è stata sottolineata in una eccellente recensione di Vladimiro Valerio (*Spunti e osservazioni dal libro di Lucio Russo L'America dimenticata. I rapporti tra le civiltà e un errore di Tolomeo*, in "Geostorie", Anno XXI – nn. 1-2, 2013, pp. 77-97). È uno dei molti studiosi che giustamente si sono rivoltati contro le argomentazioni capziose e parziali portate avanti dal Russo, di solito acuto ed informatissimo esegeta di tanti personaggi e momenti della storia della scienza.

Leggere la scheda di Valerio è doveroso ed illuminante; non ci sarebbe molto da aggiungere alle sue considerazioni critiche. Ma per ampliare il quadro delle analisi che smantellano l'impalcatura discorsiva di Russo consiglio di consultare il saggio di Dmitry A. Shcheglov, *The Accuracy of Ancient Cartography Reassessed: The Longitude Error in Ptolemy's Map* ("Isis", vol. 107, n. 4, 2016, pp. 687-706). Per avvalorare le proprie deduzioni, Lucio Russo dà per scontato che Tolomeo abbia adoperato una metrologia con lo stadio di m. 157,5. Questa è solamente un'arbitraria assunzione che non trova conforto nelle fonti letterarie e nell'archeologia. "In recent years a number of researchers—Dennis Rawlins (Baltimore), Lucio Russo (Rome), Irina Tupikova (Dresden), and Klaus Geus (Berlin)—have independently proposed essentially the same hypothesis. They all suggest that the longitudinal stretching of Ptolemy's map can be explained completely by a single factor: using the wrong value for the circumference of the Earth. This essay presents a critical response to this hypothesis. The hypothesis proceeds from a remarkable coincidence between the average stretching factor of Ptolemy's map (see above) and the ratio between the two best known ancient estimates of the circumference of the Earth: 252,000 stades calculated by Eratosthenes (ca. 276–194 b.c.e.) and 180,000 stades adopted by Ptolemy—namely, 252/180 = 1.4. This coincidence means that if we place Ptolemy's map, or any part of it, with its linear distances remaining unchanged, onto a sphere with Eratosthenes' circumference, then all its outlines will turn out to be pretty accurate in comparison with modern maps (numerous examples have been discussed by Tupikova and Geus).

Meanwhile, there was a crucial difference between Eratosthenes' and Ptolemy's estimates. Whereas the former was almost universally accepted in antiquity, including by Ptolemy himself in his main lifework, the *Almagest*, the latter had been almost unknown before Ptolemy accepted it in the Geography, one of his later treatises. It is reasonable to assume, therefore, that early versions of Ptolemy's map or its sources were also initially based on Eratosthenes' value. On the basis of these observations, it has been argued that the east–west stretching of Ptolemy's map was caused by a change in the proposed value of the Earth's circumference. This means that all the linear distances (in stades, miles, etc.) underlying his map had initially been supposed to be measured on a sphere with Eratosthenes' circumference, but eventually were converted to angular units (degrees) on a much smaller sphere. As a result, all the distances when expressed in angular units increased by a factor of 1.4. This hypothesis implies that Ptolemy's sources—or, to put it differently, the early version of his map—were uncannily accurate in terms of spherical coordinates. From this fact, the advocates of the hypothesis draw two far-reaching conclusions. The first is that distance measurements underlying Ptolemy's map and Eratosthenes' value for the circumference of the Earth were equally accurate. This, in turn, implies that the length of the stade used by Eratosthenes must have been close to the 1/252,000 part of the actual circumference of the Earth—that is, close to 158.8–159 m. The latter conclusion finds strong support in a long-standing and widely accepted view that Eratosthenes and many other Greek authors of the pre-Roman period used a stade of about 157.5 m, instead of the 185 m accepted after the Roman conquest. When using these "short" stades, Eratosthenes' value for the Earth's circumference works out to be 39,690 m—that is, an error of less than 1 percent. Similarly, many other distances quoted by ancient sources become surprisingly accurate when expressed in the "short" stades [...] In fact, at the very basis of the "wrong circumference hypothesis" there lies a patently false deduction. It is true that Gossellin, Russo, Tupikova, and Geus have convincingly demonstrated that Ptolemy's coordinates, recalculated for a sphere with Eratosthenes' circumference, turn out to be highly accurate. But the further conclusions that they draw from this observation do not necessarily follow. First of all, the match between the recalculated Ptolemy coordinates and the modern ones, however close, does not in itself mean that the longitudinal distortion in Ptolemy's map was due entirely to a single cause—namely, the wrong value for the Earth's circumference. Nor does it warrant discarding two other possible causes: the exaggeration of distances and the lengthening of the stade. Another crucial point to stress is that Ptolemy's recalculated map turns out to be accurate only in terms of spherical coordinates. This does not mean that the actual distance measurements underlying it and Eratosthenes'

value for the Earth's circumference were equally accurate. In fact, we cannot ascertain the accuracy of these two factors separately, but only the accuracy of their quotient (that is to say, the accuracy of distances converted into degrees). This uncertainty suggests an alternative and more plausible explanation for the spectacular accuracy of Ptolemy's coordinates recalculated for Eratosthenes' Earth, as I shall show below. Finally, all three main points of the hypothesis in question—the incorrect value for the Earth's circumference as the only cause of the stretching of Ptolemy's map, the high accuracy of ancient distance measurements, and the use of the "short" stade (which is implied by the high accuracy of Eratosthenes' value for the circumference)—conflict with the evidence from other sources. And the conflict is so serious that either it entails a radical revision of our conventional views on ancient geography or (as is more likely) destroys the hypothesis itself. First, there are good reasons to conclude that both Ptolemy and Eratosthenes used stades close to 185 m in length, which was commonly accepted in the Roman period, whereas the "short" stade of 157.5 m seems rather like a scholarly fiction (see Sects. III–V). Expressed in these "common" stades, Eratosthenes' value for the Earth's circumference turns out to be overestimated by 16.5 percent (252,000 x 185 m = 46,620 km) compared with the true size, whereas Ptolemy's value proves to be underestimated by 16.7 percent (180,000 x 185 m = 33,300 km). Second, it is highly doubtful that Greek geographers ever had really accurate distance measurements at their disposal. Both of our main authorities on ancient geography, Ptolemy and Strabo, emphasize repeatedly that distances reported by travelers are considerably exaggerated: by a third or even by half, according to Ptolemy, and Strabo's estimates are similar. More thorough comparison with modern data shows that on average ancient distances were overestimated by approximately 20–25 percent. These results have an interesting corollary: if overestimated distances are expressed in degrees of the almost equally overestimated great circle of Eratosthenes' Earth, then the two errors cancel each other. A map constructed on this basis turns out to be quite accurate in terms of spherical coordinates. This explains why Ptolemy's map, placed on Eratosthenes' Earth, matches modern maps remarkably closely. If the same distances, overestimated by at least 20 percent, are converted to degrees according to Ptolemy's value for the Earth's size, which was underestimated by about 17 percent, then the resulting map becomes stretched by approximately 40 percent in angular terms [...] To conclude, it seems doubtful that all distances recorded by ancient sources, even in the Roman period, could always have been measured in the same standard stades. Mere doubt, however, is not enough to outweigh concrete evidence. Meanwhile, the evidence allows us to draw quite definite conclusions. First, if the length of the stade did vary, it most probably varied within the range between the

extreme attested values, 165–213 m. Second, if an average length of the stade, or the most widely used one, is to be found, then the “common” stade of 185 m is the most plausible candidate for that role. Finally, the most evident conclusion is that the “short” stade of 157.5 m is nowhere attested: we have neither reliable written evidence for it, nor stadiums of this length, nor feet equal to 1/600 of this stade (which would equal ca. 262.5 mm) [...] The idea that Eratosthenes and some other Hellenistic geographers used a “short” stade is both the cornerstone and the Achilles heel of the “wrong circumference hypothesis.” This idea has been repeatedly criticized and rejected as unfounded, but it is still revived time and again, with enviable persistence. I hope this article can contribute to sorting the wheat from the chaff in this controversy. The starting point for any discussion about a “short” stade, and the only direct evidence for it, is a brief remark in Pliny (*Natural History* 12.53) that “by the calculation of Eratosthenes, a schoenus measures 40 stades, that is 5 miles, but some authorities have made the schoenus 32 stades.”55 It may seem clear that Pliny is talking here about the well-known “common” stade equal to ⅛ of a Roman mile. But the advocates of the “short stade hypothesis” discard Pliny’s reference to the mile and try to determine Eratosthenes’ stade through the schoenus. The Greek term σχοῖνος literally means “rush” or “a rope twisted of rushes” and refers, primarily, to a local Egyptian unit of distance but also to similar units used in Mesopotamia and Persia. Our main sources emphasize that the schoenus, unlike the stade, was a very vague measure:56 it could vary fourfold, from 30 to 120 stades.57 Therefore, it would be reasonable to define the schoenus through the stade, as Pliny clearly does in the quoted passage, and not vice versa.58 But this has not stopped the advocates of the “short stade hypothesis.” They make the following calculations. One schoenus is most often defined as 30 stades. One stade traditionally consists of 400 cubits. The royal Egyptian cubit has been firmly determined on the basis of archaeological evidence as circa 525 mm.59 Therefore, one schoenus is equal to 6,300 m, while Eratosthenes’ stade, equal to 1/40 of a schoenus, works out at 157.5 m. This result, with minor variations, was first suggested by P. S. Girard, J. A. Letronne, and Friedrich Hultsch60 and remains very popular to this day.61 These considerations suffice to show how precarious is the proposed interpretation of Pliny’s remark: it is based on a combination of different testimonies, none of which is reliable enough by itself. This construction can hardly be regarded as a firm basis for far-reaching conclusions about Eratosthenes’ stade, nor can it alone outweigh ample evidence for the common use of the 185-m stade [...] I have examined more than 160 distances given in Pliny’s Natural History (ca. 79 c.e.) and obtained the following results. If these distances are believed to be accurate, then, by the same logic that has led us to the “itinerary stade,” we have either to conclude that Pliny’s mile was equal to

circa 1,190 m, which is impossible, or to assume that Roman measurements of distances were much less accurate than Greek ones, which is hard to believe. Otherwise, we have to conclude that Pliny's distances were overestimated by 25 percent on average—that is, by approximately the same amount that Eratosthenes' and Strabo's distances must have been if they were expressed in the "common" stades of 185 m. This argument, of course, needs further elaboration and testing, but in any case it seems to undermine the "short stade hypothesis." It remains to add that, as applied to Eratosthenes' measurement of the Earth, the idea of the "itinerary stade" creates an obvious vicious circle. If the "short" stade is deduced from a tacit assumption of highly accurate ancient distance measurements, then it is unsurprising that, when substituted into Eratosthenes' calculations, it makes his result amazingly accurate as well [...] I try to show that the interplay of different factors behind the longitudinal deformation of Ptolemy's map was even more complicated than it may appear from what has been said so far. In spite of the criticisms of the "wrong circumference hypothesis" I have presented, its core assumption that the early version of Ptolemy's map was based on Eratosthenes' value for the circumference of the Earth receives strong confirmation from a different side. Thus, a number of researchers—Ernst Herzfeld, Antonin Wurm, and I—have independently demonstrated that a large part of Ptolemy's map represents with high accuracy the main parameters of Eratosthenes' geographical system translated into degrees according to Eratosthenes' scale (where 1° of the meridian is 700 stades, and not 500 as in Ptolemy). Let us consider this point briefly. The framework of Eratosthenes' "world map" was based on the system of so-called sphragides or "seals": large regions shaped like quadrangles bounded by parallels and meridians. Strabo, our sole source on this subject, describes only four sphragides in Asia. In the north all of them were bounded by the Taurus range. In the west and east they were delimited by meridians: one passed along the Indus and separated India from Ariana; another passed through the Caspian Gates (the Sirdar Pass in Iran) and separated Ariana from the third sphragis. The western boundary of the third sphragis was the Euphrates. Already, at first sight, the outlines of the second and the third sphragides are clearly visible on Ptolemy's map (see Figure 2). More important, about twenty distances given by Eratosthenes for this area fit closely with Ptolemy's map (they are shown by the arrows on Figure 2). As an illustration, I will consider only the two most important examples concerning the longitudinal extent of these sphragides. The length of Ariana (from the Caspian Gates to the Indus) was, according to Eratosthenes, 14,000 stades at about the latitude of Rhodes (36º on Ptolemy's map). In Ptolemy, Ariana lies between the meridians of 94º (through the Caspian Gates) and 119º (the boundary of India), which equals about 14,000 stades at

the latitude of Rhodes (where 1º long = 700 stades x cos 36º = 566 ≈ 560 stades). The length of the third sphragis from Babylon to the eastern boundary is stated to be 9,000 stades at about the latitude of Alexandria (31º on Ptolemy's map). In Ptolemy, the longitudinal interval between Babylon (79º) and the eastern boundary of the sphragis (94º) is precisely 9,000 stades at the latitude of 31º (where 1º long = 700 stades x cos 31º = 600 stades).

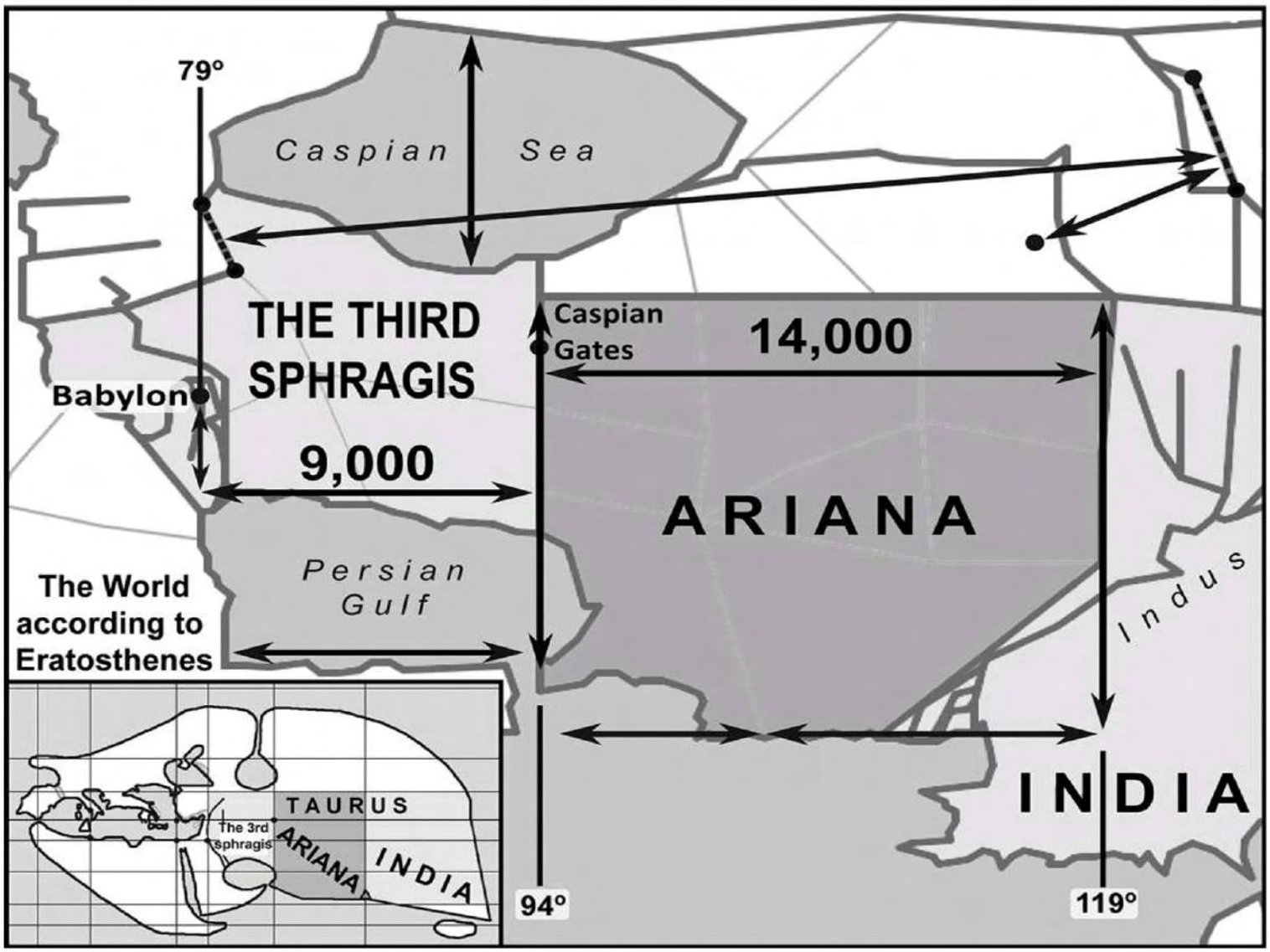

Figure 2. Eratosthenes' distances underlying Ptolemy's map.

These and other similar observations suggest that the whole area of Ptolemy's map, at least between Babylon and the Indus, has been based on a set of distances expressed in Eratosthenes' degrees. This, on the one hand, confirms that the early version of Ptolemy's map may have been based on Eratosthenes' value for the Earth's circumference. But, on the other hand, it undermines the idea that Ptolemy's adoption of a smaller value was the main factor responsible for the stretching of his map: a part of his map was not recalculated to the 500-stade degrees but nevertheless has almost the same

stretching factor as the other parts [...] The results of the investigation are twofold. On the one hand, we have merely reaffirmed, albeit on a more solid footing, an opinion more than two centuries old. It may be summarized as follows. Ancient surveyors usually overestimated distances, which was inevitable for that time. It is hardly surprising that Eratosthenes' measurement of the Earth, based on one of these distances, came to be overestimated as well. Interestingly, the errors in Eratosthenes' and Ptolemy's estimates of the Earth's circumference were almost equal in magnitude, but opposite in sign. Consequently, distances projected on Ptolemy's Earth become still more exaggerated in angular terms than they were to begin with. Therefore, only about half of Ptolemy's error in longitude can be attributed to his incorrect value for the Earth's size. In contrast, when projected on Eratosthenes' Earth the same distances come to be quite accurate in angular terms. The incredible accuracy of the reconstructed early Ptolemaic map, therefore, proves to be a quaint illusion produced by a superposition of two opposite errors. In these circumstances, it would be unfair to blame Ptolemy for his errors, because the whole tradition he relied on was a chain of errors. On the other hand, two other observations are more important. First, we have defined the range in which the length of the Greek stade could vary and, therefore, the extent to which this variation could have contributed to Ptolemy's cartographic errors. Second, we have found that a large part of Ptolemy's map was expressed in degrees according to Eratosthenes' rate of 1º = 700 stades, but nevertheless it exhibits the same longitudinal error as other parts. The combination of all these factors must inevitably have produced a rather confused picture".

Dmitry Shcheglov ha condotto una serrata ed ineccepibile indagine che mina alle fondamenta la credibilità delle conclusioni raggiunte dal Russo. Nel mio piccolo desidero fare una breve sosta sulla sconsiderata proposta di traslocare le Isole Fortunate o dei Beati (punto zero per il calcolo della longitudine tolemaica) nelle Piccole Antille (Isole Sopravengo e Sottovento). Questo il passaggio cardine Russo: "In Russo 2013a [*Ptolemy's longitudes and Eratosthenes' measurement of the Earth's circumference*, "Mathematics and Mechanics of Complex Systems", 1, pp. 67–79] the longitudes reported by Ptolemy were used to determine the "effective" value of Ptolemy's stadium. The main result of that work was an independent validation of the substantial exactness of the value 157.5 deduced from Pliny, which implies that the estimate for the Earth's circumference by Ptolemy was very wrong. That paper was based on the analysis of longitudes of a sample of 80 cities, chosen as the most renowned of the part of the world best known in Hellenistic times. Plotting Ptolemy's longitudes vs. the values currently accepted for them, a graph was obtained which is approximated remarkably well by means of a linear regression. The equation of the regression line is:

$$y=1.428x+17.06 \qquad (1)$$

and the coefficient of determination is $R2=0.9935$ [...] A further confirmation of the above reconstruction comes from a comparison between the global regression coefficient (1.428) and the ratio between the actual longitudinal amplitude for the Ptolemy's oikoumene and the value of 180° accepted by him. The westernmost locations considered by Ptolemy are the Islands of the Blessed, which he identified with the Canary Islands. In order to estimate the global dilation of the oikoumene, on the other extreme we consider the Sera Metropolis (today Xi'an), since more eastern locations are not unanimously identified today. Since the actual longitudes of Xi'an and the Canary Islands are respectively 108°54' E and 15° W, and the corresponding values given by Ptolemy are 177°15' and 0°, the ratio between their difference of longitude given by Ptolemy and the corresponding actual value is approximately 1.43, and thus very close to the regression coefficient given in Russo 2013a [...] We conjectured that the value of 180° for the amplitude of the *oikoumene*, i.e. the difference in longitude between the Islands of the Blessed and the easternmost regions, was taken from a Hellenistic source (most probably Hipparchus, who may well have intended it as a rounded value). On the other hand we know that Hellenistic scientists, and in particular Hipparchus, had accepted the value of Eratosthenes for the Earth's circumference (Strabo, *Geographica*, I, iv, 1; II, v, 7; II, v, 34), while the smaller estimate, grossly wrong, was first introduced (as far as we know) by Posidonius (Strabo, *Geographica*, II, ii, 2). Why this value of 180°, which according to Hipparchus was consistent with the measurement by Eratosthenes, coexists with a much "smaller" Earth in (among others) Ptolemy? Since there is a substantial agreement in the distances along the circles of latitude between Ptolemy's and Eratosthenes' data, it is clear that the only possibility is that there was a misinterpretation on the identification of one of the two extrema of the oikoumene. Two arguments allow us to exclude that the issue involved locations in the far East:

1. It is *a priori* easier to misinterpret the identification of oceanic islands, for which no close locations are known for very large distances, than that of a city which is reachable through a series of intermediate locations.
2. To produce an error of the order of magnitude of that made by Ptolemy, the original location of some town in China assumed by his source as the eastern extremum of the oikoumene had to be very far into the Pacific Ocean. The only remaining possibility, therefore, is that the two errors were originated by an erroneous identification of the Islands of the Blessed. The previous reasoning, which up to now was intended at analyzing the origin of the errors in Ptolemy's *Geographia*, can also disclose new possibilities in a more general (and possibly more relevant) issue. We are indeed led to the conclusion that the Islands of the Blessed, to which the Hellenistic source of Ptolemy (most probably Hipparchus) referred, can be found approximately on

the opposite semi-meridian with respect to the more eastern locations cited in the *Geographia*. This entails the need to consider longitudes that are much farther on the west than those believed as known in Hellenistic times. We arrived at this conclusion by elimination, after having considered and discarded every possible alternative. Therefore, in my opinion, we should seriously consider the extreme consequences of the previous reasoning. As a renowned investigator used to say, *when you have eliminated the impossible, whatever remains, however improbable, must be the truth*. The elimination process illustrated before conduced us to the conclusion that the first cause for the errors made by Ptolemy was the wrong identification of the Islands of the Blessed with the Canary Islands. One may wonder how was it possible such a misinterpretation of the sources. It is therefore useful, in my opinion, to briefly recall some key historical processes which played a relevant role in this connection. The mentions of travels in the Atlantic Ocean are not negligible in ancient sources. However, between Hellenistic age and Ptolemy, a relevant loss of geographic knowledge concerning this Ocean occurred, and in a short time even well known descriptions of voyages started to be considered unreliable. The report of the famous expedition made by Pytheas, for instance, in which he described the iced sea and the midnight sun, was considered trustworthy by Hellenistic scientists such as Eratosthenes and Hipparchus, but was later rejected by Strabo (*Geographica*, II, iv, 1; II, iv, 2; II, iii, 5), was not copied and transmitted anymore, and finally lost. The error made by Ptolemy according to our reconstruction was made possible by the combination of the loss of knowledge about the Atlantic Ocean with a general weakening of science in the Roman world with respect to the Hellenistic age. Geography, in particular, was transformed radically, from the mathematically founded subject it was in Hellenistic times to the purely descriptive one it became in works like those by Strabo and Polybius. An error like the misidentification of an archipelago became clearly much more probable once the use of spherical coordinates (latitude and longitude) for the identification of the locations had been abandoned, as it was in the first century BC. The oldest evidence concerning the identification of the Islands of the Blessed with the Canaries dates indeed from the first century BC, and then was inherited by Ptolemy when he attempted at reconstructing quantitatively the mathematical geography for the first time after the methodological crisis had occurred. Posidonius seems the most probable candidate as the source of the misidentification. Indeed he is to our knowledge the most ancient source giving the length of 500 stadia per degree for the Earth's circumference, which tend to suggest that the transition from the "ancient" to the "new" value occurred within his work. A quantitative analysis of the data given by Ptolemy strongly supports the idea of a mistaken identification of the archipelago. Indeed:

1. The latitude he gives for the Islands is wrong by about 15°, an enormous error which can be regarded as incompatible with any data coming from real measures and can be only explained as the result of a confusion between two different archipelagos.

2. The Canaries are spread over a total longitude of about four and a half degrees, while the archipelago considered by Ptolemy covers just one degree in west-east direction; moreover, the Canary Islands cover less than two degrees in north-south direction, while Ptolemy's Islands five and a half. In conclusion, the two archipelagos have both a strip-like shape, but they are approximately oriented in orthogonal directions. A decisive test in order to check the proposed thesis is at this point possible. We can indeed compute the original longitude of the Islands of the Blessed simply taking a difference of longitude of 180° with respect of Ptolemy's eastern extreme locations, and verify whether we come close to some archipelago. The longitude of the eastern bound of Ptolemy's oikoumene can be computed by the regression line (1) given above, solving the equation:

$$180 = 1.428\,x + 17.06$$

In this way, we eventually get the following coordinates of the Islands:
65°54'W, 16°N
64°54'W, 15°15'N
65°54'W, 14°15'N
65°54'W, 12°30'N
64°54'W, 11°N
65°54'W, 10°30'N
These locations are graphically represented in Fig. 1:

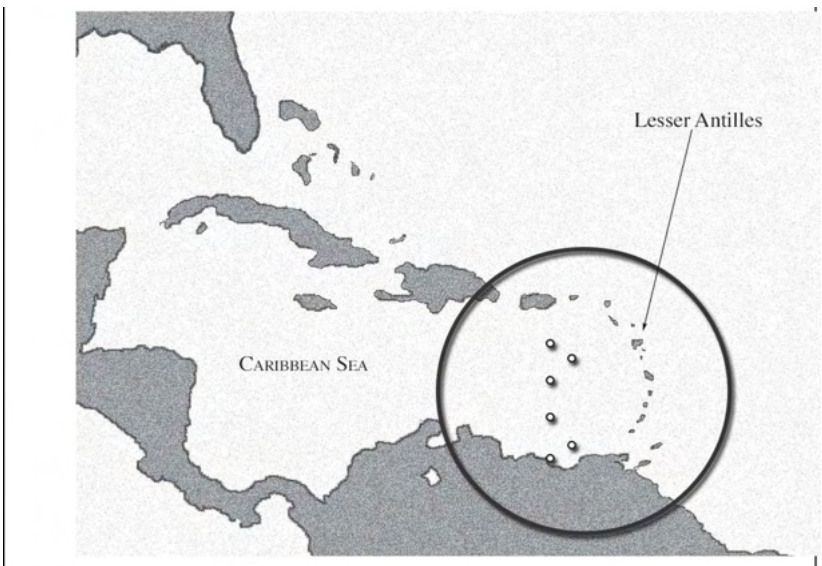

Fig. 1 – The circlets indicate the original locations of the Island as computed in the text.

As the reader can see, not only we come very close to an actually existing archipelago (formed by the Leeward Islands and the Windward Islands, in the

Lesser Antilles), but also its dimension, shape and orientation correspond well to the dots [...] The following passage by Pliny has eluded, until now, any reasonable interpretation: Regarding the length and the breadth this is what I deem worthy of mention. For the whole circumference, Eratosthenes, a man highly regarded by all and surpassing others in every subtlety of learning, and especially in the present matter, gave the value of 252,000 stadia. [...] Hipparchus, a man to be admired for taking issue with him and for much more besides, then added to that number a little less than 26,000 stadia [De longitude ac latitude haec sunt, quae digna memoratu putem. Universum autem circuitum Eratosthenes, in omnium quidem literarum subtilitate, et in hac utique praeter ceteros sollers, quem cunctis probari video, CCLII milium stadiorium prodidit, [...] Hipparchus, et in coarguendo eo, et in reliqua omni diligentia mirus, adicit stadiorum paulo minus XXVI milia (Piny, *Naturalis Historia*, II, 247)]. The last statement by Pliny is contradicted by the passages by Strabo in which it is told that Hipparchus accepted the value given by Eratosthenes for the Earth's circumference (*Geographica*, I, iv, 1; II, v, 7; II, v, 34). The key in order to understand the previous passage, in my opinion, is contained in its first words. Since it makes no sense to talk about the length and the breadth of a spherical object such as the Earth, and since immediately before Pliny was discussing the dimensions of the oikoumene, it is very plausible that the subject of the source (misunderstood by Pliny, who had in mind the most celebrated measurement by Eratosthenes) was not changed, since it was usual in geographical works to give length and breadth of the *oikoumene*. We indeed know that Eratosthenes, besides the circumference of the Earth, had also calculated the length of the *oikoumene* along the parallel of Athens, getting the result of 77,800 stadia, of which 5,000 at the west of the Pillars of Heracles (Strabo, *Geographica*, I, iv, 5). The passage can therefore be explained conjecturing that Hipparchus had extended in longitude the *oikoumene* by adding almost 26,000 stadia to the value given by Eratosthenes. Since this extension could hardly have affected the Pacific Ocean, we have to conclude that the western bound of the oikoumene according to Hipparchus was at 31,000 (26,000 + 5000) stadia from the Pillars of Heracles along the parallel of Athens. Since the cosine of the latitude of Athens is about 0.788, a degree of longitude along this parallel, according to Eratosthenes, corresponds to about 700 × 0.788 = 552 stadia, which implies that we reach a location which is approximately 56°10' at the west of Gibraltar, corresponding to a longitude of 61°31'W. The corresponding semi-meridian is graphically represented in Fig. 2, and represents a striking confirmation of the thesis that Hipparchus knew the coordinates of the Leeward and Windward Islands, and also of the correctness of our reconstruction of the meaning of the passage by Pliny. Moreover, it indirectly supports the idea that the source of Ptolemy on the

Islands of the Blessed was Hipparchus. We notice that it is not surprising that the match between theoretical previsions and actual geographical data is in Fig. 2 much better than in Fig. 1, since in that case the reconstruction had a statistical basis and relied on the value of 180° for the amplitude of the oikoumene, which was possibly rounded, while in this case the method only uses one quantitative datum taken from the sources. Of course, the addition of 26,000 stadia by Hipparchus has to be considered not a correction of the value given by Eratosthenes, but rather an updating due to new geographical discoveries" (*Far-reaching Hellenistic geographical knowledge cit*., pp. 2-17).

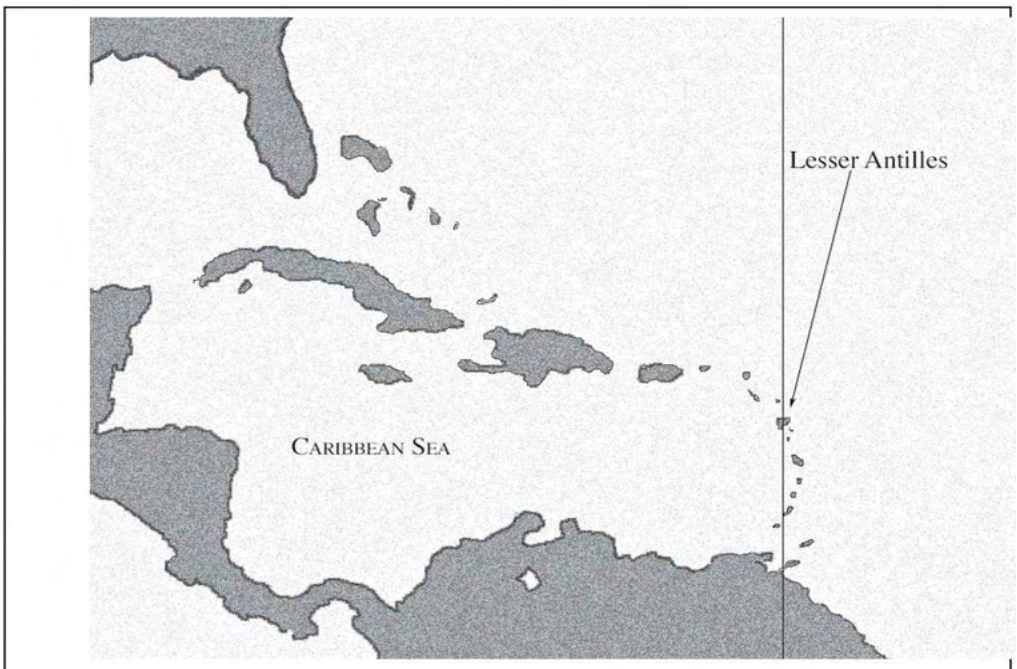

Fig. 2 – the westernmost longitude of the *oikoumene* according Hipparchus, as recovered by the Pliny's passage.

Ora, mi sembra impossibile che un fisico e matematico come Lucio Russo si perda in un bicchiere d'acqua. Ne prendo solo un sorso per dire che il risultato dell'equazione lineare della retta di regressione (1) è 114,10, ossia 114°6'E, che grossomodo corrispondono ai 109° di Xi'an (*Metropolis Sinae*) posta da Tolomeo all'estremità orientale (180°) dell'*oikoumene* numerabile, anche se non del mondo: καὶ ἡ μητρόπολις Σίναι ἣ Θίναι οπ γό νότ. γ οὔτε μέντοι χαλκᾶ τείχη φασὶν αὐτὴν ἔχειν, οὔτε ἄλλο τι ἀξιόλογον· περιέχεται δὲ ἀπὸ τῶν Καττιγάρων πρὸς τὰς δυσμὰς ἀγνώστῳ γῇ, περιλαμβανούσῃ τὴν Πρασώδη θάλασσαν μέχρι τοῦ Πράσου ἀκρωτηρίου, ἀφ᾽ οὗ ἄρχεται, ὡς εἴρηται, ὁ τῆς Βατραχείας θαλάσσης κόλπος, συνάπτων τὴν γῆν τῷ τε Ῥαπτῷ ἀκρωτηρίῳ καὶ νοτίοις μέρεσι τῆς Ἀζανίας (*Geogr.*, VII, 4, 5). L'identificazione di *Metropolis Sinae* (*Sera*) con Xi'an è in realtà soltanto una congettura plausibile (cfr. Alfred Stückelberger und Gerd Graßhoff, *Klaudios Ptolemaios Handbuch der Geographie*, Basel, 2006, p. 22: "von den Kanarischen Inseln bis nach Xi'an,

einer möglichen Identifizierung von Sera, sind es lediglich ca. 124°"). Secondo alcuni potrebbe essere Taiyuan, capitale della provincia di Shānxī (112°33'E).

L'errore di Russo, per riprendere il titolo del suo volume, sta nell'incongrua operazione aritmetica eseguita subito dopo (180° - 114°6' = 65°54'W). Ma in questa maniera vengono indebitamente sottratti i 180° che erano già stati trasformati nei 114°6'E mediante l'equazione della retta di regressione! Difatti, applicando questa formula alla longitudine iniziale (gli 0° di Tolomeo) si ottiene il valore decimale -11,974 pari a 11°58'O che approssimano piuttosto bene la longitudine vera dell'isola più orientale delle Canarie (Lanzarote, 13°25'O). Naturalmente il polinomio di primo grado utilizzato da Russo ha un significato statistico e per conseguire dati numerici più esatti bisogna ricorrere alla trigonometria sferica (per un approccio alternativo alle determinazioni di longitudine in Tolomeo, sempre nell'ambito delle assunzioni teoriche di sfondo condivise da Russo, cfr. Irina Tupikova & Klaus Geus, *The Circumference of the Earth and Ptolemy's World Map*, in "Max Planck Institute for the History of Science", preprint N. 439, 2013, pp. 1-26).

L'impossibile di holmesiana memoria diviene il possibile indispensabile alla tesi finale: scaltramente, Russo ha impostato le condizioni al contorno per derivare il corollario "Some source of Ptolemy's *Geographia* knew with remarkable precision the position of some locations in the Caribbean Sea, i.e. the Leeward and Windward Islands. These islands were the locations originally intended as the "Islands of the Blessed". Their misidentification with the Canaries first occurred in the first century BC and was then accepted by Ptolemy" (*Far-reaching Hellenistic geographical knowledge cit*., p. 17).

Il brano di Plinio, invece, non porta alle conseguenze tratte illecitamente da Russo. Guarda caso, questi ha omesso il resto della frase pliniana: "CCLII milium stadiorum prodidit, quae mensura Romana conputatione efficit trecentiens quindeciens centena milia passuum: inprobum ausum, verum ita subtili argumentatione conprehensum, ut pudeat non credere". Ecco il motivo:
252.000 stadi x 185 = 46.620 km.
31.500 miglia romane x 1,480 = 46.620 km.

Plinio è un testimone scomodo perché adotta la misura di 185 metri per lo stadio greco anziché i 155,6 richiesti da Russo. Non solo, ma interpretare il passo di Plinio alla stregua di una convalida formale dell'ipotesi Isole Fortunate = Piccole Antille mette Russo con le spalle al muro. Lo scrittore latino riferisce che Ipparco corresse la cifra erastotenica di 252.000 stadi per la circonferenza terrestre integrandola di ulteriori 26.000 stadi, mentre Russo pretende di impiegarla per la stima della longitudine sul parallelo di Atene dai 180° di *Metropolis Sinae* agli 0° delle Isole Sopravento e Sottovento nei Caraibi, posizione quest'ultima che Tolomeo avrebbe convertito erroneamente nella coordinata delle Canarie, accorciando di fatto il mondo di circa 56°. Siccome Russo è reticente nel trascrivere la narrazione di Plinio per

dimostrare che l'inciso su Ipparco non può che riferirsi a misure di coordinate piane invece che alle dimensioni della sfera terrestre, riproduco per esteso i paragrafi di Plinio (Nat. Hist., II, 112-113):
"Pars nostra terrarum, de qua memoro, ambienti, ut dictum est, oceano uelut innatans longissime ab ortu ad occasum patet, hoc est ab India ad Herculis columnas Gadibus sacratas, LXXXV centena LXXVIII milia p., ut Artemidoro auctori placet, ut uero Isidoro, XCVIII centena XVIII milia. Artemidorus adicit amplius a Gadibus circuitu Sacri promunturii ad promunturium Artabrum, quo longissime frons procurrat Hispaniae, DCCCCXCI milia D. Mensura currit duplici uia: a Gange amne ostioque eius, quo se in Eoum oceanum effundit, per Indiam Parthyenenque ad Myriandrum urbem Syriae in Issico sinu positam LII centena XV milia, inde proxima nauigatione Cyprum insulam, Patara Lyciae, Rhodum, Astypalaeam in Carpathio mari insulam, Taenarum Laconicae, Lilybaeum Siciliae, Caralim Sardiniae XXI centena XIII milia, deinde Gades XII centena L milia, quae mensura uniuersa ab Eoo mari efficit LXXXV centena LXXVIII milia. Alia uia, quae certior, itinere terreno maxime patet, a Gange ad Euphraten amnem LI centena LXIX milia, inde Cappadociae Mazaca CCXLIIII milia, inde per Phrygiam, Cariam, Ephesum CCCCXCVIIII milia, ab Epheso per Aegaeum pelagus Delum CC milia, Isthmum CCXII milia D, inde terra [et Laconico mari] et Corinthiaco sinu Patras Peloponnesi XC milia, Leucadem LXXXVII milia D, Corcyram totidem, Acroceraunia LXXXII milia D, Brundisium LXXXVII milia D, Romam CCCLX milia, Alpes usque ad Scingomagum uicum DXVIIII milia, per Galliam ad Pyrenaeos montes Illiberim CCCCLXVIII milia, ad oceanum et Hispaniae oram DCCCXXXI milia, traiectu Gadis VII milia D, quae mensura Artemidori ratione LXXXIX centena XLV milia efficit. Latitudo autem terrae a meridiano situ ad septentriones, dimidio fere minor Isidoro colligit LIIII centena LXII milia, quo palam fit quantum et hinc uapor abstulerit et illinc rigor; neque enim deesse terris arbitror aut non esse globi formam, sed inhabitabilia utrimque inconperta esse. Haec mensura currit a litore Aethiopici oceani, qua modo habitatur, ad Meroen DCXXV milia, inde Alexandriam XII centena L milia, Rhodum DLXXXIIII milia, Cnidum LXXXVII milia D, Coum XXV milia, Samum C milia, Chium XCIIII milia, Mytilenen LXV milia, Tenedum CXIX milia, Sigeum promunturium XII milia D, os Ponti CCCXII milia D, Carambin promunturium CCCL milia, os Maeotis CCCXII milia D, ostium Tanais CCLXXV milia, qui cursus conpendiis maris breuior fieri potest LXXIX milia. Ab ostio Tanais nihil modicum diligentissimi auctores fecere; Artemidorus ulteriora inconperta existimauit, cum circa Tanain Sarmatarum gentes degere fateretur ad septentriones uersus; Isidorus adiecit XII centena L milia usque ad Thylen, quae coniectura diuinationis est. Ego non minore quam proxime dicto spatio Sarmatarum fines nosci intellego. Et alioqui quantum esse debet quod innumerabiles gentes subinde sedem mutantes capiat? Vnde ulteriorem mensuram

inhabitabilis plagae multo esse maiorem arbitror; nam et a Germania inmensas insulas non pridem conpertas cognitum habeo. De longitudine ac latitudine haec sunt, quae digna memoratu putem. Vniuersum autem circuitum Eratosthenes, in omnium quidem litterarum subtilitate, in hac utique praeter ceteros sollers, quem cunctis probari uideo, CCLII milium stadiorum prodidit, quae mensura Romana conputatione efficit trecentiens quindeciens centena milia passuum: inprobum ausum, uerum ita subtili argumentatione conprehensum, ut pudeat non credere. Hipparchus, et in coarguendo eo et in reliqua omni diligentia mirus, adicit stadiorum paulo minus XXVI milia. Alia Dionysodoro fides; neque enim subtraham exemplum uanitatis Graecae maximum. Melius hic fuit, geometricae scientia nobilis; senecta diem obiit in patria; funus duxere ei propinquae, ad quas pertinebat hereditas. Hae cum secutis diebus iusta peragerent, inuenisse dicuntur in sepulcro epistulam Dionysodori nomine ad superos scriptam: peruenisse eum a sepulcro ad infimam terram; esse eo stadiorum XLII milia. Nec defuere geometrae qui interpretarentur, significare epistulam a medio terrarum orbe missam, quod deorsum ab summo longissimum esset spatium et idem pilae medium. Ex quo consecuta conputatio est, ut circuitum esse CCLII milia stadiorum pronuntiarent. Harmonica ratio, quae cogit rerum naturam sibi ipsam congruere, addit huic mensurae stadiorum XII milia terramque XCVI partem totius mundi facit".

A me pare che lo stacco tra la parte che riguarda latitudini e longitudini e quella che concerne la misura della circonferenza della Terra non comporti necessariamente un'incomprensione della fonte da parte di Plinio che avrebbe introdotto il discorso per lui più familiare della famosa misurazione di Eratostene del meridiano terrestre. Anche perché il prosieguo del testo con l'episodio della lettera di Dionisodoro conferma il mutamento del focus narrativo, virato dalle determinazioni di superficie dell'*oikoumene* alle dimensioni del globo intero. "Nach einem S., das 1/40 Schoinos, also 1/10 Meile betrug, hat nach Plin. n. h. XII 53 u. a. Eratosthenes gerechnet, und ein S., welches 10mal in der Meile enthalten ist, finden wir auch in Italien angewendet (Strab. V 3, 12 p. 239) und, dazu genau stimmend, in Syrien (Mommsen Herm. III 429ff.) eine Fußlänge von 1/6000 römische Meile, also von 5/6 römische Fuß (u. S. 1942). Auf den Parasangen (Schoinos), rund und mindestens 5940 m, kommen also 40 dieser S. Das 30fache dieses Stadiums (= 3/4 Parasang, rund und min- destens 4455 m) lebt in der lieue de France (4452 m) nach (unt. S. 1946 sub 8). Die Erkenntnis, daß dieses das S. ist, welches Eratosthenes, der natürlich auch andere S. kannte und auf sie Bezug genommen hat, seiner Gradmessung zugrunde legte eine Erkenntnis, zu der Hultsch (§ 41, 6, vgl. § 9, 3), obgleich er richtig auf die Verwendung eines S. von 1/40 Schoinos schloß, nicht gelangen konnte, weil er das Wesen des Schoinos als eines mit dem Parasangen identischen Maßes nicht erkannte,

ist von hervorragender Bedeutung. Denn es ergibt sich daraus, daß Eratosthenes' Berechnung ungleich genauer ausgefallen war, als man bisher annahm. Wenn nämlich Eratosthenes den Erdgrad auf 700 S. berechnete, so war dieses Resultat nicht um 1/6 (162/30) zu groß, wie es scheinen mußte, wenn man die S. irrigerweise als Achtelmeilen-S. ansah (Censorin. 13, 2 *nam ut Erato- sthenes geometrica ratione collegit maximum terrae circuitum esse stadiorum ducentum quinquaginta duum milium, ita Pythagoras quot stadia inter terram et singulas stellas essent indicavit. sta- dium autem in hac mundi mensura* usf., vgl. o. S. 1934, 15), auch nicht um 3/25 (12%) zu groß, wie anzunehmen wäre, wenn Eratosthenes hier nach dem sonst auch von ihm verwendeten (Iulianus von Askalon, Metr. script. I 201) sta- dium Olympicum (81/3 auf die Meile, oben sub 2) gerechnet hätte, sondern das Ergebnis war um ungefähr 1/15 zu klein, blieb also nur um einen verhältnismäßig geringen Bruchteil hinter dem wahren Betrage zurück: 700 Zehntelmeilen-S. sind siebzig Meilen. 1 Meile beträgt 5000 römische Fuß; der Fuß normal mindestens 297 mm; die Meile also mindestens 1485 m; 70 Meilen 103,950 km. Der Aquatorialgrad aber beträgt 15 geographische Meilen zu je 7,420438 km, also 111,30657 km: Differenz 7,35657 km, also nur ganz wenig mehr als 1/15 des wahren Betrages. Die Differenz wird noch geringer (ca. 2/31), wenn man gemäß dem wahrscheinlichsten Betrag (Congr. 241 [77] ff., o. S. 1932 sub e und S. 1945) des olympisch- attisch-römischen Fußes (297,7 mm) die Meile auf 1488,5 m ansetzt, in welchem Falle sich die 70 Meilen auf 104,195 km berechnen. Wenn daher Plin. n. h. II 247 angibt, Hipparch habe zu der eratosthenischen Berechnung des Erdumfangs auf 252 000 S. noch etwas weniger als 26 000 S., also mehr als 1/10 des Ganzen gefügt, so wurde dadurch eine bedeutende Korrektur erzielt, die allerdings in etwas (ca. 1/30) über das Ziel hinausschoß. Die Erkenntnis von dem Vorhandensein dieses Zehntelmeilenstadiums und die richtige Würdigung von Eratosthenes' mit ihm operierender Erdmessung ist nun freilich in alter wie in neuester Zeit durch eine Reihe von Irrtümern und Mißverständnissen erschwert, ja geradezu unmöglich gemacht worden" (*Paulys Realencyclopädie der classischen Altertumswissenschaft*, *s.v. Stadion2*).

Tuttavia cfr. il commento al brano di Plinio su Ipparco e Dionisodoro di D. R. Dicks: "This remark of Pliny's cannot be reconciled with the earlier testimony of Strabo. The latter explicitly states in frags. 35, 36 and 39 that Hipparchus used Eratosthenes' figure of 252,000 stades for the circumference of the earth, and nowhere even hints that this figure was increased. Berger (*Hipp.*, p. 27) gives various suggestions that have been made to explain Pliny's statement; viz. that Hipparchus added 26,000 stades on the basis of a better drawn meridian, or that, as he defended the old maps against Eratosthenes' corrections, so he took the figure of 278,000 stades as a mean between Eratosthenes' figure and an old measurement of 300,000

stades mentioned by Archimedes in his 'Sandreckoner' (Aren. i, sect. 8), but with no indication of who made it or how (it is perhaps to be attributed to Dicaearchus, a pupil of Aristotle; see Thomson, pp. 153-5). However, none of this is satisfactory in view of Strabo's clear testimony. Lehmann-Haupt (s.v. 'Stadion' in P-W, cited above), in attempting to prove that Eratosthenes used the stade of to to the Roman mile (here also he relies heavily on Pliny's statement to this effect, *Nat. Hist*. xii, 53) and therefore his earth measurement was only about one-fifteenth too small instead of about one-sixth too large (as on the eight-to-a-mile stade), dogmatically accepts Pliny's single assertion about the addition made by Hipparchus, and states that it was a true correction and in fact brought the result to slightly more than the correct figure. Lehmann- Haupt admits that other sources contradict Pliny, but apparently has such faith in Pliny's evidence that he does not consider the other testi- mony as worthy of any credence at all; he clenches his arguments by stating that since the ten to the mile stade gives a result closer to the true figure, then it must have been used! Such reasoning is hardly convincing. D. J. Campbell (*C. Plini Secundi liber secundus*, Aberdeen Univ. Studies, No. 118, p. 91) notes that there is an abrupt change of subject here - Pliny has been giving various distances in the *οίκουμένη* and then suddenly switches to the circumference of the whole earth - which lends weight to Berger's own suggestion that the 26,000 stades results from the difference between the 63,000 stades which make one · *τεταρτημόριον* or "quarter" according to Eratosthenes' division of the total circumference into four quarters (Str. end of 113), and the 38,000 stades which Eratosthenes gives for the breadth (north-south) of the inhabited world (Str. 63), the figures being misunderstood and completely confused by Pliny. This is plausible; but in view of the fact that he follows the statement with the absurd story of Dionysodorus' letter, and that his mathematical and geographical knowledge is very shaky, no great reliance can be placed on his testimony on this subject, and one is forced to the conclusion, in agreement with Gossellin (*La géogr. des Grecs analysée* (1790), p. 53) and most modern commentators, that this remark is merely an egregious blunder" (*The Geographical Fragments of Hipparchus*, Londin, 1960, p. 153). "Nel pensier mi fingo" Tolomeo sulle spalle di Ipparco che sbarca alle Piccole Antille...

Analisi matematica e geodetica della collocazione delle Isole Fortunate in Lucio Russo: una revisione critica avanzata

Abstract

Quest'appendice fornisce un'analisi matematica e geodetica avanzata della procedura utilizzata da Lucio Russo per ricollocare il meridiano zero della *Geographia* di Tolomeo dalle Isole Canarie alle Piccole Antille. Si dimostra che la regressione lineare globale tra longitudini moderne e tolemaiche non costituisce una trasformazione affine valida su S^1, non è invertibile in senso statistico, non è stabile sotto estrapolazione e non è compatibile con la struttura geodetica delle coordinate sferiche. Si mostra inoltre che la sottrazione di 180° applicata da Russo non ha alcuna giustificazione geometrica su una sfera, e che l'applicazione corretta della regressione al valore $y = 0°$ restituisce una longitudine moderna compatibile con le Canarie. Vengono analizzati anche la variabilità regionale dei coefficienti, la metrologia dello stadio, la plausibilità dell'identificazione della *Metropolis Sinae* e la debolezza del passo pliniano invocato come conferma indipendente.

1 Introduzione

Le longitudini della *Geographia* di Tolomeo presentano una dilatazione sistematica rispetto ai valori reali. Russo (2013, 2018) propone di sfruttare tale distorsione per ricostruire la posizione reale del meridiano zero tolemaico, giungendo alla conclusione che esso corrisponderebbe alle Piccole Antille. Questa monografia mostra che tale deduzione non è supportata né dalla matematica della regressione lineare, né dalla geodesia sferica, né dalla metrologia antica, né dalla filologia.

2 Modello matematico delle longitudini tolemaiche

Tolomeo misura le longitudini rispetto al meridiano delle Isole Fortunate. Le longitudini moderne sono misurate rispetto a Greenwich. Sia:

$$x_i \in [-180^\circ, 180^\circ] \quad \text{(moderne)}, \qquad y_i \in [0^\circ, 180^\circ] \quad \text{(tolemaiche)}.$$

Russo assume un modello lineare globale:

$$y = ax + b.$$

Tale modello è matematicamente problematico perché:

- x e y sono variabili angolari su S^1, non variabili reali su $\mathbb{R}$;
- la relazione tra due sistemi di longitudini è una trasformazione di gruppo su S^1;
- una trasformazione affine non rispetta la struttura ciclica di S^1.

3 Regressione lineare su variabili angolari

La regressione OLS minimizza:

$$\sum_i (y_i - (ax_i + b))^2.$$

Ma su S^1:

$$y_i \sim y_i + 360^\circ,$$

e la distanza naturale è:

$$d(\theta_1, \theta_2) = \min_{k \in \mathbb{Z}} |\theta_1 - \theta_2 + 360^\circ k|.$$

La regressione lineare ignora completamente questa struttura.
Inoltre:

$$\text{OLS non è simmetrica:} \quad \text{OLS}(x \to y) \neq \text{OLS}(y \to x)^{-1}.$$

Quindi l'inversione algebrica della retta è matematicamente illegittima.

4 Regressione su S^1: problemi strutturali

Su S^1, le trasformazioni ammissibili sono:

$$\theta' = \theta + \Delta\theta \mod 360^\circ.$$

Una trasformazione affine:

$$\theta' = a\theta + b$$

non preserva:

- la periodicità,
- la distanza geodetica,
- la struttura di gruppo.

Quindi non può rappresentare una relazione tra sistemi di longitudini.

5 Inversione della regressione: impossibilità formale

L'inversione:

$$x = \frac{y - b}{a}$$

non è una regressione inversa.
La regressione inversa corretta richiederebbe:

$$\min_{a',b'} \sum_i (x_i - (a' y_i + b'))^2,$$

che produce valori diversi.
L'inversione algebrica è quindi priva di significato statistico.

6 Geodesia sferica: differenze di meridiano e antipodi

La distanza geodetica tra due punti è:

$$d = R \arccos(\sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos(\Delta\lambda)).$$

La differenza di longitudine $\Delta\lambda$ non determina la distanza.
L'operazione di Russo:

$$x_{\text{ovest}} = x_{\text{est}} - 180^\circ$$

implica antipodalità, che non è garantita.

7 Analisi dell'estrapolazione a 180°

Il valore $y = 180^\circ$ è fuori dal dominio dei dati usati per la regressione.

L'errore standard cresce in modo non lineare con la distanza dal centro del campione. L'estrapolazione è quindi instabile e non affidabile.

8 Perché la sottrazione di $180°$ è geodeticamente illegittima

Su una sfera:

$$\lambda_{\text{ovest}} = \lambda_{\text{est}} - 180°$$

è valido solo per punti antipodali.

Tolomeo non colloca la *Metropolis Sinae* in posizione antipodale rispetto alle Isole Fortunate.

La sottrazione è quindi priva di fondamento geodetico.

9 Analisi statistica avanzata

9.1 OLS

Minimizza errori verticali.

9.2 TLS

Minimizza errori ortogonali.

9.3 ODR

Modello più corretto per dati con errori in entrambe le variabili.

Nessuno di questi modelli giustifica l'inversione della retta di Russo.

10 Variabilità regionale dei coefficienti

Shcheglov mostra che i coefficienti locali variano enormemente:

Regione	Pendenza	R^2
Mediterraneo occidentale	1.77–1.81	1
Mediterraneo orientale	1.15–1.19	1
Media globale	1.428	0.987

La pendenza globale è una media di distorsioni eterogenee.

11 Metrologia dello stadio

Lo stadio "corto" da 157.5 m non è attestato.

Lo stadio romano da 185 m produce errori che si compensano naturalmente.

12 Identificazione della *Metropolis Sinae*

L'identificazione con Xi'an è congetturale.

Spostare questo punto di un grado sposta il presunto meridiano occidentale di un grado.

13 Analisi filologica del passo di Plinio

Il passo è considerato un errore da Dicks, Gossellin e altri.

Non può fungere da conferma indipendente.

14 Conclusioni

La regressione globale:

- non è una trasformazione valida su S^1;
- non è invertibile;
- non è stabile sotto estrapolazione;
- applicata correttamente a $y = 0°$ restituisce le Canarie;
- non giustifica lo spostamento del meridiano zero nelle Piccole Antille.

Riferimenti

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La retta di regressione di Russo e la localizzazione delle Isole Fortunate. Parte II

Nel paper mi sono espresso in modo alquanto impreciso parlando di "indebita sottrazione di 180 gradi" dalla formula di regressione. In questa seconda parte dell'appendice discuto in modo meno tecnico e da un'altra angolazione l'errore logico e storico di Russo sulla longitudine tolemaica delle Isole Fortunate.

1 Il contesto: la dilatazione delle longitudini in Tolomeo

È assodato che la *Geographia* di Claudio Tolomeo presenta una sistematica dilatazione delle differenze di longitudine rispetto ai valori reali. Il fattore medio di dilatazione è circa 1.4, valore non casuale: coincide con il rapporto fra la misura eratostenica della circonferenza terrestre (252 000 stadi, 700 stadi/grado) e quella adottata da Tolomeo su suggerimento di Posidonio (180 000 stadi, 500 stadi/grado):

$$\frac{252}{180} = 1.4.$$

Russo (2013; 2018) ha analizzato un campione di 80 città ellenistiche confrontando le longitudini tolemaiche y con quelle reali moderne x, ottenendo la retta:

$$y = 1.428\,x + 17.06 \qquad (R^2 = 0.9935). \tag{1}$$

Shcheglov (2014), su 245 punti, ottiene:

$$y = 1.4279\,x + 16.425 \qquad (R^2 = 0.9874). \tag{2}$$

L'alta bontà di adattamento è reale, ma non implica la conclusione che Russo ne ricava.

2 Il presupposto nascosto della retta di regressione

La retta è costruita assegnando a ciascuna città la sua longitudine reale x e la corrispondente longitudine tolemaica y, misurata dal meridiano zero di Tolomeo. Questo meridiano zero è, per definizione, quello delle Isole Fortunate (0°), identificate con le Canarie almeno da Pomponio Mela in poi. Le Canarie si trovano a circa −15°.

Ne consegue che il termine noto +17.06 codifica esattamente lo scarto fra lo 0° tolemaico e lo 0° moderno. La retta è dunque costruita assumendo che le Canarie siano il meridiano zero: non può essere usata per dimostrare che il meridiano zero fosse altrove.

Formalmente: se C è la proposizione "le Canarie sono il meridiano zero", la retta è costruita assumendo C. Usarla per negare C equivale a dedurre una negazione dalle premesse che la contengono già.

3 L'errore aritmetico: i due 180° non sono la stessa grandezza

3.1 La catena di calcolo di Russo

Russo applica la retta all'estremo orientale dell'ecumene, la *Metropolis Sinae*:

$$180^\circ = 1.428\,x + 17.06 \quad \Rightarrow \quad x = \frac{180 - 17.06}{1.428} \approx 114.1^\circ\text{E}.$$

Fin qui il calcolo è corretto. Russo poi sottrae un secondo 180°:

$$\lambda_{\text{ovest}} = x - 180^\circ \approx -66^\circ.$$

Questa sottrazione è aritmeticamente ingiustificata.

3.2 I due sistemi non sono compatibili

Grandezza	**Sistema**	**Significato**
$y = 180^\circ$	Tolemaico	Longitudine di Xi'an nel catalogo
180° sottratti	Reale moderno	Metà della circonferenza terrestre

Il primo 180° è una coordinata tolemaica; il secondo è un'ampiezza reale. Sottrarli come se fossero omogenei è privo di significato geometrico.

Con la retta di Shcheglov si ottiene:

$$x = \frac{180 - 16.425}{1.4279} \approx 114.5^\circ\text{E}, \qquad \lambda_{\text{ovest}} \approx -65.5^\circ.$$

La sensibilità mostra che il risultato non è stabile.

4 La dimostrazione diretta: la retta restituisce le Canarie

L'unico uso corretto della retta per localizzare il meridiano zero è porre $y = 0^\circ$:

$$0 = 1.428\,x + 17.06 \quad \Rightarrow \quad x = -\frac{17.06}{1.428} \approx -11.95^\circ.$$

Lanzarote si trova a $-13^\circ 25'$, perfettamente compatibile.

Con Shcheglov:

$$x = -\frac{16.425}{1.4279} \approx -11.50^\circ.$$

Entrambe le rette indicano le Canarie, non i Caraibi.

5 Il coefficiente regionale: un argomento contro la causa unica

Shcheglov suddivide il campione in sottoinsiemi geografici:

Regione	Coefficiente	R^2
Mediterraneo occidentale	1.77–1.81	≈ 1
Mediterraneo orientale	1.15–1.19	≈ 1
Media globale	1.428	0.987

La variazione da 1.15 a 1.81 è incompatibile con una singola causa globale.

6 Il problema dello stadio

La tesi di Russo richiede uno stadio corto di 157.5 m, derivato da Plinio. Shcheglov (2016) mostra:

- nessuna attestazione diretta dello stadio corto;
- la derivazione è circolare;
- lo stadio comune romano era 185 m.

Plinio stesso converte 252 000 stadi in 31 500 miglia romane, implicando 185 m.

7 L'identificazione della Metropolis Sinae

L'identificazione con Xi'an è solo congetturale (Stückelberger & Graßhoff 2006). Varianti di pochi gradi spostano di conseguenza il presunto punto caraibico.

8 Il passo di Plinio come "conferma indipendente"

Russo interpreta i 26 000 stadi come estensione dell'ecumene verso occidente. Ma:

- Strabone afferma che Ipparco accettò la misura di Eratostene;
- il contesto pliniano è confuso;
- i commentatori moderni considerano il passo un errore.

9 Tavola di sintesi

Argomento	Difetto	Esito
Retta di regressione	Circolarità	Invalido
Sottrazione dei 180°	Incoerenza aritmetica	Invalido
Applicazione a $y = 0°$	Uso corretto	Conferma Canarie
Stadio corto	Premessa non provata	Debole
Plinio II, 247	Testo errato	Fragile

10 Conclusioni

L'argomento centrale di Russo è viziato da tre difetti concatenati: circolarità logica, incoerenza aritmetica e uso improprio della regressione. Applicando correttamente la retta si ottengono le Canarie, non i Caraibi. A ciò si aggiungono la fragilità dello stadio corto, l'incertezza sulla *Metropolis Sinae* e la debolezza del passo pliniano.

Le coordinate tolemaiche partono dalle Canarie, e la regressione di Russo — correttamente interrogata — lo conferma.